Homework: 6, 7, 11, 20, 27, 29, 54, 58, 66

6. An electron's position is given by $\vec{r} = 3.00t\hat{i} - 4.00t^2\hat{j} + 2.00\hat{k}$ with t in seconds and $\vec{r}$ in meters. (a) In unit-vector notation, what is the electron's velocity $\vec{v}(t)$? At t=3.00 s, what is $\vec{v}$ (b) in unit vector notation and as (c) a magnitude and (d) an angle relative to the positive direction of the x axis?

(a) $$\vec{v}(t) = \frac{d\vec{r}}{dt} = \frac{d}{dt}(3.00t\hat{i} - 4.00t^2\hat{j} + 2.00\hat{k})$$

$$\vec{v}(t) = 3.00\hat{i} - 8.00t\hat{j}$$

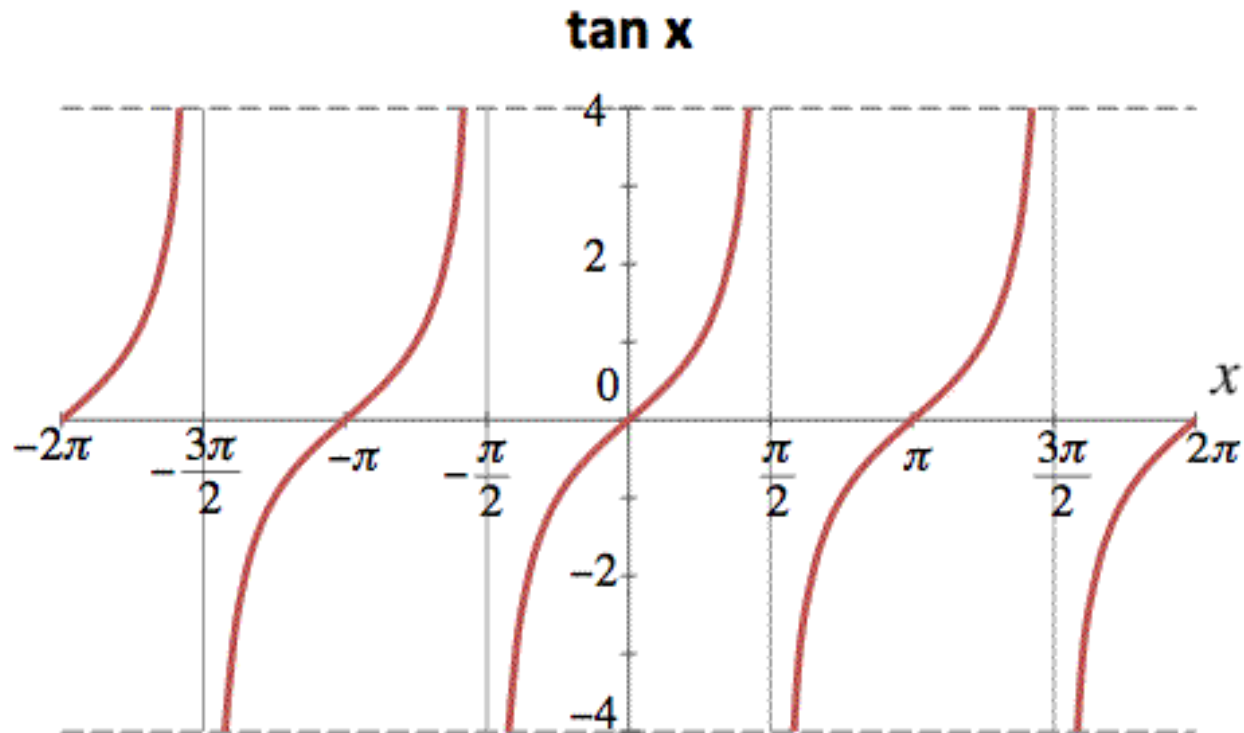(b) at t = 3s: $$\vec{v} = 3.00\hat{i} - 24.00\hat{j}$$

(c) the magnitude of the velocity:

$$v = \sqrt{3.00^2 + (-24.00)^2} = 24.2 \ (m/s)$$

(d)
$$\tan\theta = \frac{v_y}{v_x} \Rightarrow \theta = \tan^{-1}\left(\frac{-24}{3}\right)$$

$$\theta_1 = -82.9^0 \quad \text{or} \quad \theta_2 = \theta_1 + 180^0 = -82.9^0 + 180^0 = 97.1^0$$

We choose $\theta = -82.9^0$ (the fourth quadrant) since $v_x > 0$ and $v_y < 0$

tan x

11. The position $\vec{r}$ of a particle moving an xy plane is given by $\vec{r} = (2.0t^3 - 5.0t)\hat{i} + (6.0 - 7.0t^4)\hat{j}$, with r in meter and t in second. In unit-vector notation, calculate (a) $\vec{r}$, (b) $\vec{v}$, and (c) $\vec{a}$ for t=2.0 s. (d) What is the angle between the positive direction of the x axis and a line tangent to the particle's path at t = 2.0 s?

(a) at t = 2 s:

$$\vec{r} = (6m)\hat{i} - (106m)\hat{j}$$

(b)

$$\vec{v} = \frac{d\vec{r}}{dt} = (6.0t^2 - 5.0)\hat{i} - (28.0t^3)\hat{j}$$

So, at t = 2 s:

$$\vec{v} = (19m/s)\hat{i} - (224m/s)\hat{j}$$

(c)

$$\vec{a} = \frac{d\vec{v}}{dt} = (12.0t)\hat{i} - (84.0t^2)\hat{j}$$

So, at t = 2 s:
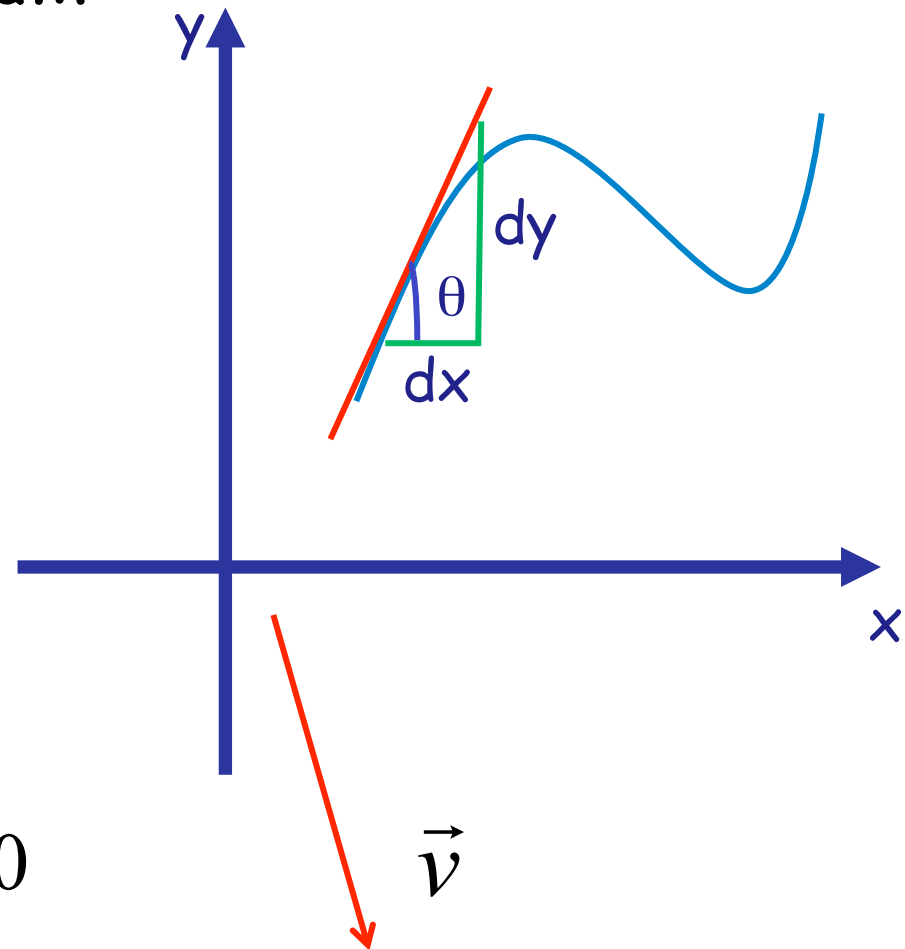
$$\vec{a} = (24m/s^2)\hat{i} - (336m/s^2)\hat{j}$$

(d) a line tangent to the particle's path:

$$\tan\theta = \frac{dy}{dx} = \frac{\dfrac{dy}{dt}}{\dfrac{dx}{dt}} = \frac{v_y}{v_x}$$

$$\tan\theta = -\frac{224}{19}$$

$$\Rightarrow \theta_1 = -85.2^0; \theta_2 = 94.8^0$$

$V_y<0$, $V_x>0$, so the velocity should be in the fourth quadrant

27. A certain airplane has a speed of 290 km/h and is diving at an angle of θ = 30⁰ below the horizontal when the pilot releases a radar decoy. The horizontal distance between the release point and the point where the decoy strikes the ground is d = 700 m. (a) How long is the decoy in the air? (b) How high was the release point?

This is a projectile motion

$$v_x = v\cos\theta$$

$$v_y = v\sin\theta$$

$$v = 290km/h = 80.6m/s$$
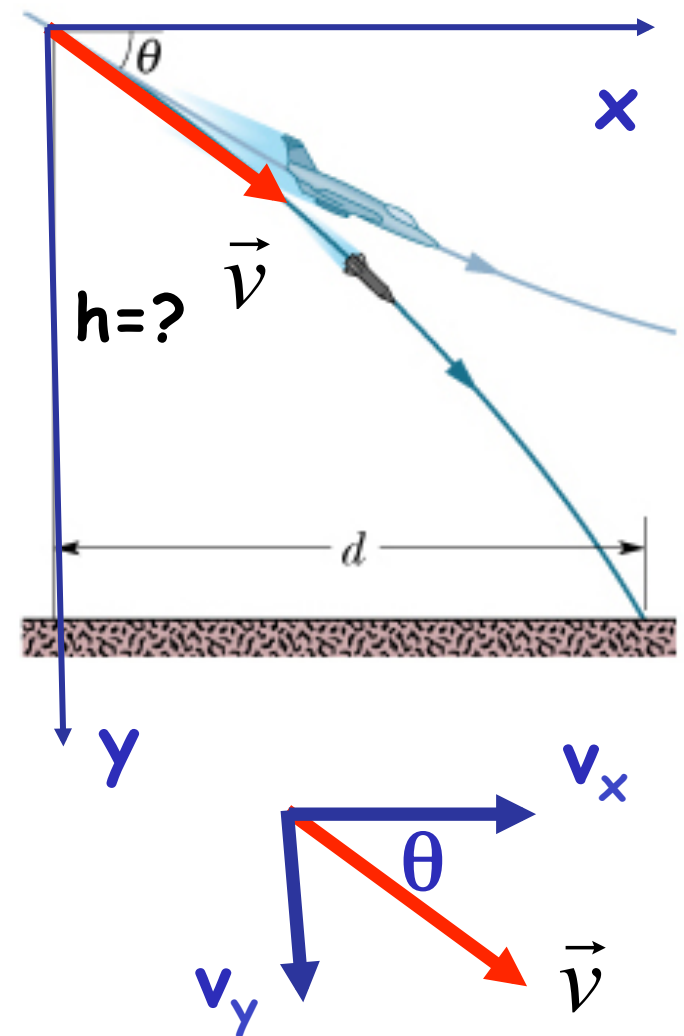
(a) Motion along the x axis:

$$t = \frac{d}{v\cos\theta} = \frac{700}{80.6\times0.866} \approx 10 \text{ (s)}$$

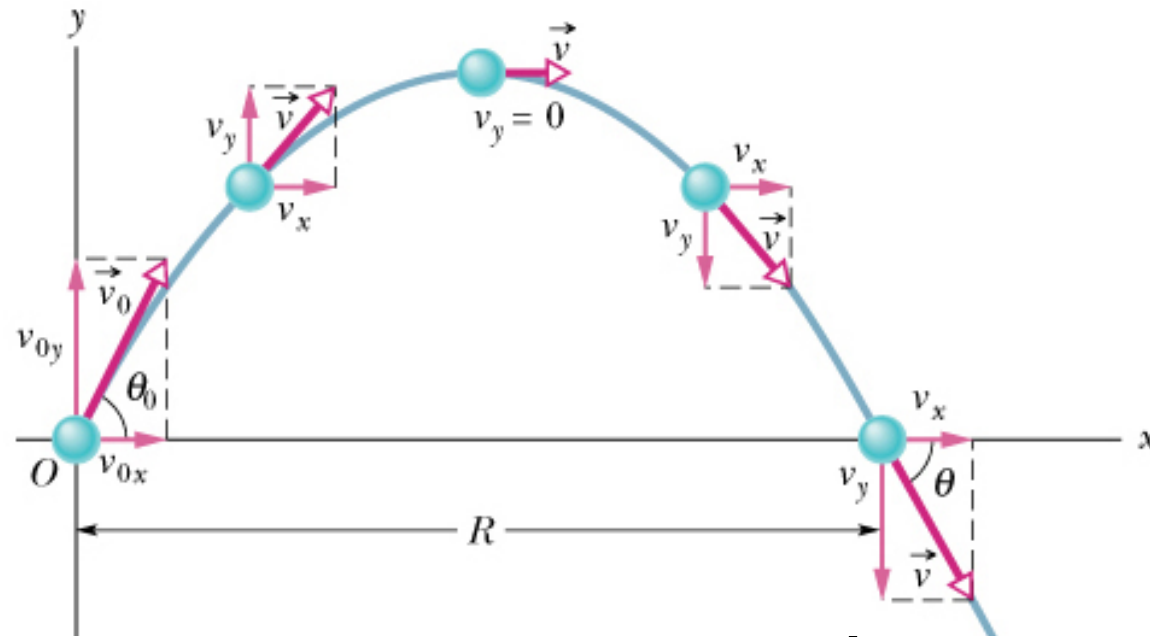**(b) We have:**

$$y = y_0 + v_{0y}t + \frac{1}{2}gt^2$$

$$h = y = (v\sin\theta)t + \frac{1}{2}gt^2$$

$$h = 893(m)$$

29. A projectile's launch speed is five times its speed at maximum height. Find launch angle θ_0.

- At maximum height: $v_y=0$; $v=v_x=v_{0x}$

  We have $v_0 = 5\ v_{0x}$

$$\cos(\theta_0) = \frac{V_{0x}}{V_0} = \frac{1}{5}$$

$$\Rightarrow \theta_0 = 78.5^0$$

66. A particle moves along a circular path over a horizontal xy coordinate system, at constant speed. At time t_1=5.0 s, it is at point (5.0 m, 6.0 m) with velocity (3.0 m/s) $\hat{j}$ and acceleration in the positive x direction. At time t_2=10.0 s, it has velocity (-3.0 m/s) $\hat{i}$ and acceleration in the positive y direction. What are the (a) x and (b) y coordinates of the center of the circular path if t_2-t_1 is less than one period?

This is a uniform circular motion
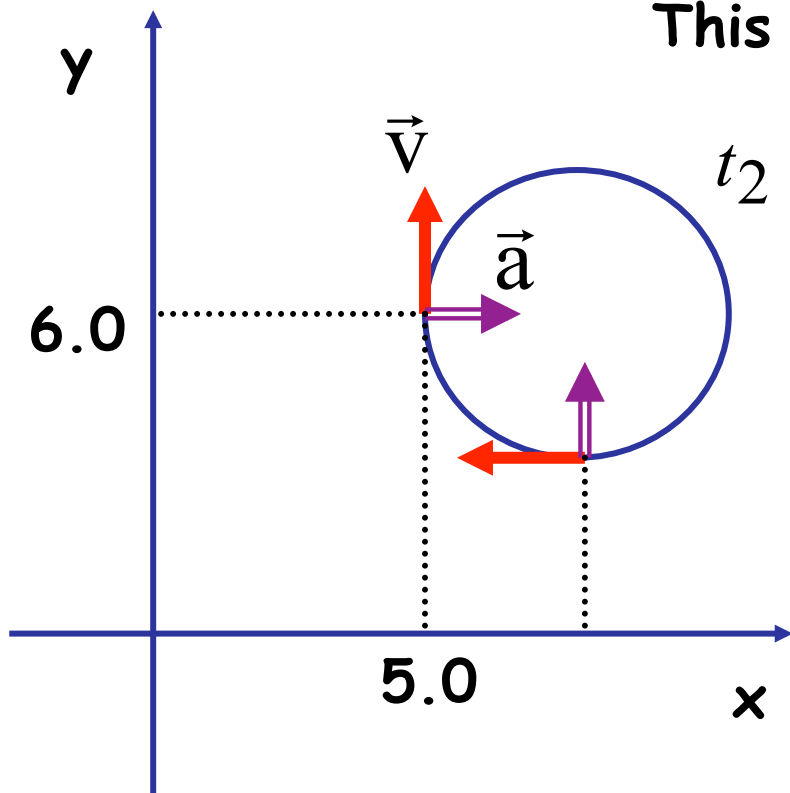
$$t_2 - t_1 = \frac{3}{4}T + nT = 5.0\,(s)\,(n = 0 \text{ as } t_2 - t_1 < T)$$

$$T = 6.67\,(s)$$

$$T = \frac{2\pi r}{v} \Rightarrow r = \frac{Tv}{2\pi} = 3.18\,(m)$$

$$x_{center} = 5.0 + 3.18 = 8.18\,(m)$$
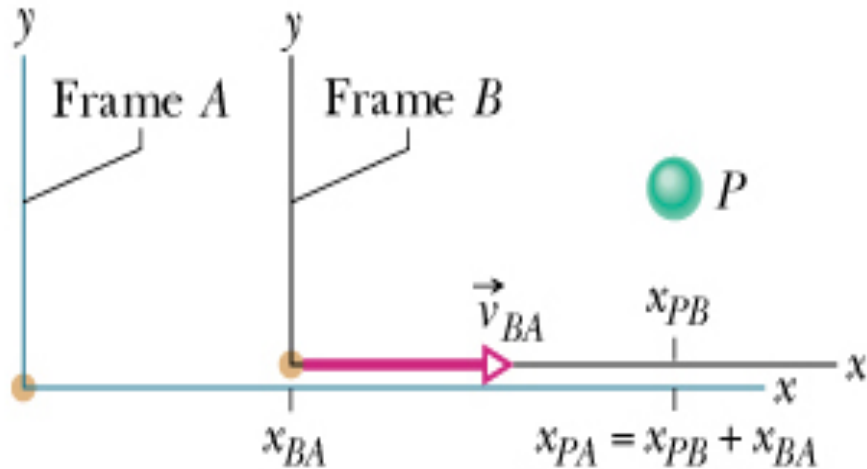
$$y_{center} = 6.0\,(m)$$

1.2.4. Relative Velocity and Relative Acceleration

A. In one dimension:

- An is parked, watching a car P speed past; Bao is driving at constant speed and also watching P:

$$X_{PA} = X_{PB} + X_{BA}$$

$$\frac{d(x_{PA})}{dt} = \frac{d(x_{PB})}{dt} + \frac{d(x_{BA})}{dt}$$

$$v_{PA} = v_{PB} + v_{BA}$$

➜ The velocity v_{PA} of P as measured by A is equal to the velocity v_{PB} of P as measured by B plus the velocity v_{BA} of B as measured by A.

If car P is moving with an acceleration: $\dfrac{d(v_{PA})}{dt} = \dfrac{d(v_{PB})}{dt} + \dfrac{d(v_{BA})}{dt}$

$$v_{BA} = a \text{ constant}: \quad a_{PA} = a_{PB}$$

➜ Observers on different frames of reference that move at constant velocity relative to each other will measure the same acceleration for a moving object.

B. In two dimensions:

$$\vec{r}_{PA} = \vec{r}_{PB} + \vec{r}_{BA}$$

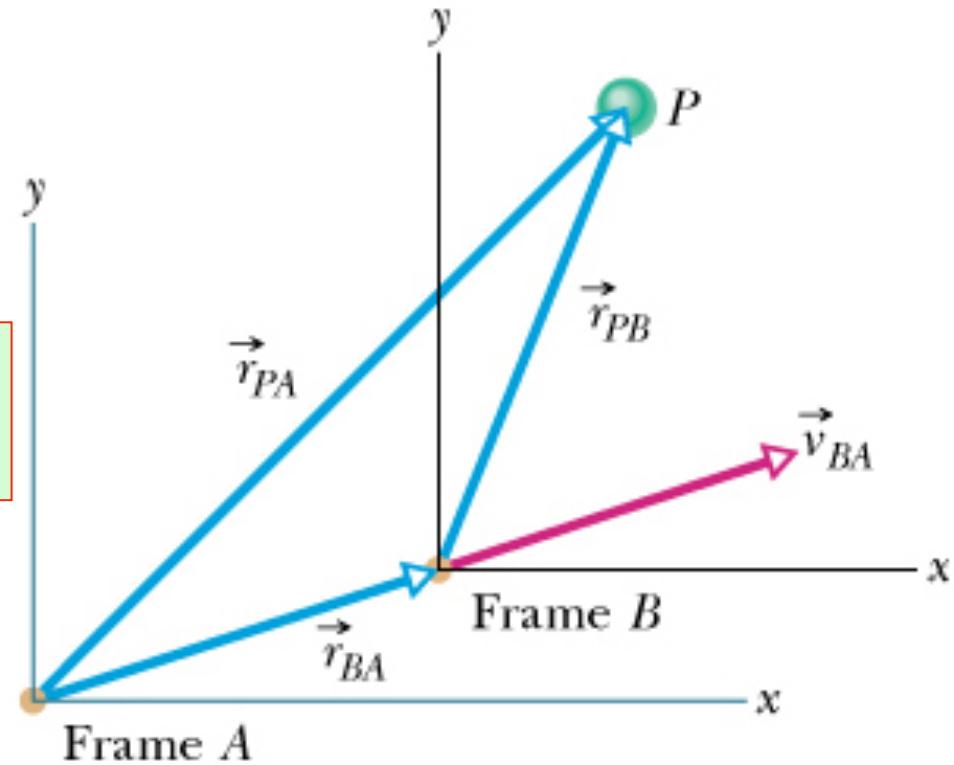$$\Rightarrow \vec{v}_{PA} = \vec{v}_{PB} + \vec{v}_{BA}$$

$$\frac{d(\vec{v}_{PA})}{dt} = \frac{d(\vec{v}_{PB})}{dt} + \frac{d(\vec{v}_{BA})}{dt}$$
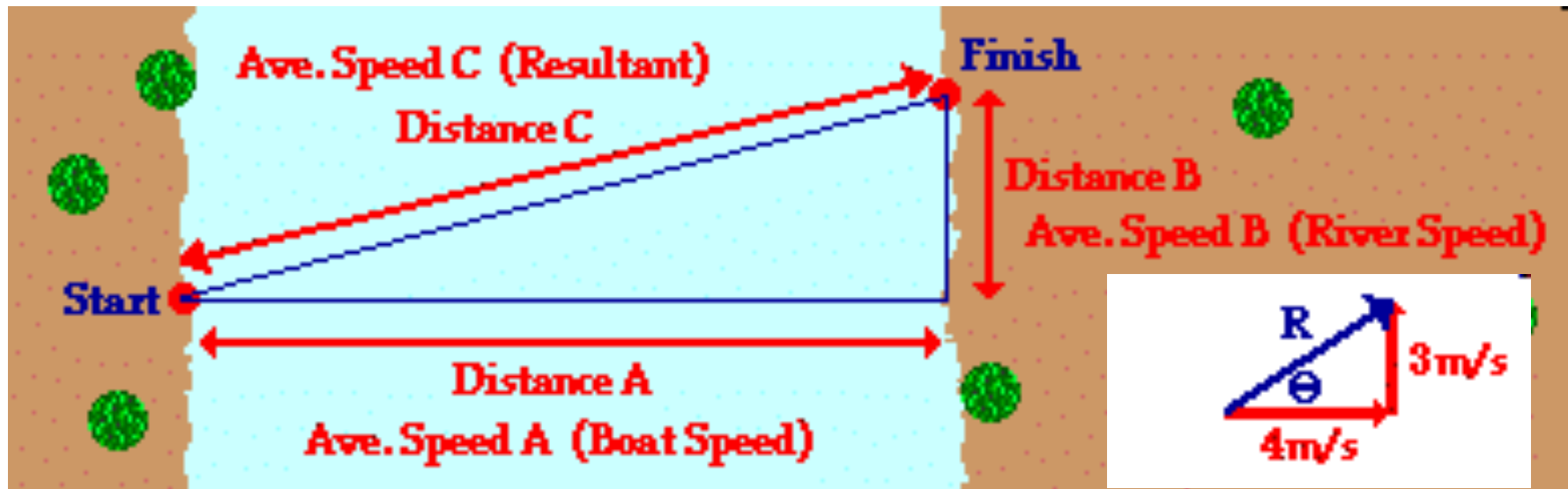
$$\Rightarrow \vec{a}_{PA} = \vec{a}_{PB}$$

Note:

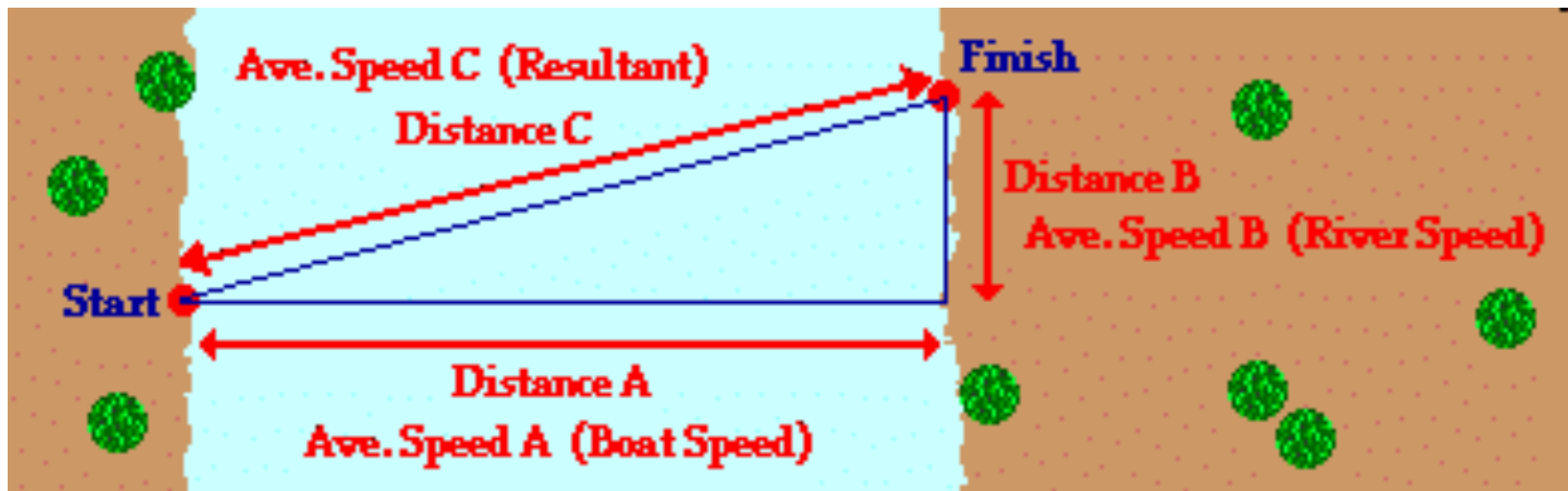$$\vec{v}_{AB} = -\vec{v}_{BA}$$

Homework: 70, 76 (page 82, 83)

Example: A motorboat traveling 4 m/s, East encounters a current traveling 3.0 m/s, North. What is the resultant velocity of the motorboat? If the width of the river is 80 meters wide, then how much time does it take the boat to travel shore to shore? What distance downstream does the boat reach the opposite shore?

$$\vec{V}_{boat/shore} = \vec{V}_{boat/river} + \vec{V}_{river/shore}$$

$$V_{boat/shore} = \sqrt{V^2_{boat/river} + V^2_{river/shore}} = R$$

$$R = 5 \text{ (m/s)}; \tan\theta = \frac{3}{4} \Rightarrow \theta = 36.9^0$$

Ave. Speed C (Resultant)
Distance C

Finish

Distance B
Ave. Speed B (River Speed)

Start

Distance A
Ave. Speed A (Boat Speed)

$$R = 5 \text{ (m/s)}; \tan\theta = \frac{3}{4} \Rightarrow \theta = 36.9^0$$

time to cross the river:

$$t = \frac{distanceA}{v_{boat/river}} = \frac{distanceB}{v_{river/shore}} = \frac{distanceC}{v_{boat/shore}}$$

$$t = \frac{distanceA}{v_{boat/river}} = \frac{80}{4} = 20 \text{ (s)}$$

distance downstream:

$$distanceB = v_{river/shore}t = 3 \times 20 = 60 \text{ (m)}$$

Chapter 2 Force and Motion

2.1. Newton's First Law and Inertial Frames

2.2. Newton's Second Law

2.3. Some Particular Forces. The Gravitational Force and Weight

2.4. Newton's Third Law

2.5. Friction and Properties of Friction.
       Motion in the Presence of Resistive Forces

2.6. Uniform Circular Motion and Non-uniform Circular Motion
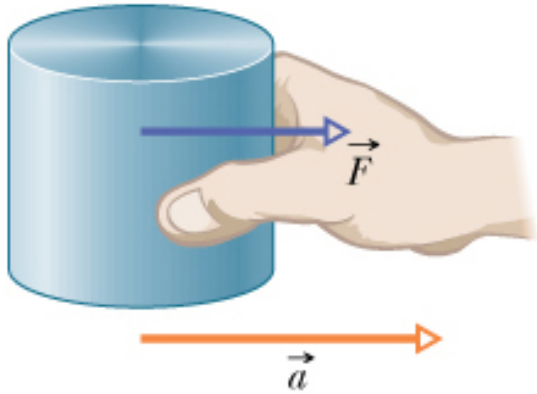
2.5. Motion in Accelerated Frames

Newtonian Mechanics

• The study of the relation between a force and the acceleration it causes is called *Newtonian Mechanics* based on Newton's three laws of motion.

• If the speed of objects is large, comparable to the speed of light, Newtonian mechanics is replaced by Einstein's special theory of relativity.

• If the size of objects is comparable to the atomic scale, Newtonian mechanics is replaced by quantum mechanics.

2.1. Newton's First Law and Inertial Frames

Force (N): Loosely speaking, force is a push or pull on an object.

If no net force acts on a body, the body's velocity cannot change; that is, the body cannot accelerate.

$$\vec{F} = 0 \quad \text{or} \quad \sum_{i=1}^{n} \vec{F_i} = 0$$

Inertial Reference Frames: (Inertial Frames) A reference frame in which there are no accelerations, only zero or uniform motion in a straight line. In other words, an inertial frame is one in which Newton's laws hold.

Example: The ground is an inertial frame if we neglect Earth's astronomical motions (such as its rotation, precession).

Checkpoint 1: Which of the figure's six arrangements correctly show the vector addition of forces $\vec{F_1}$ and $\vec{F_2}$ to yield the third vector, which is meant to represent their net force $\vec{F}_{net}$?
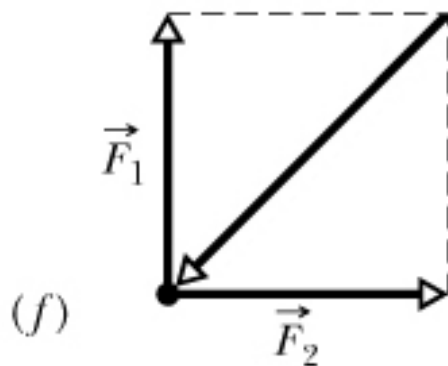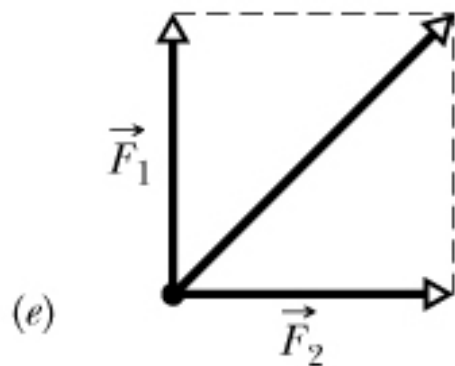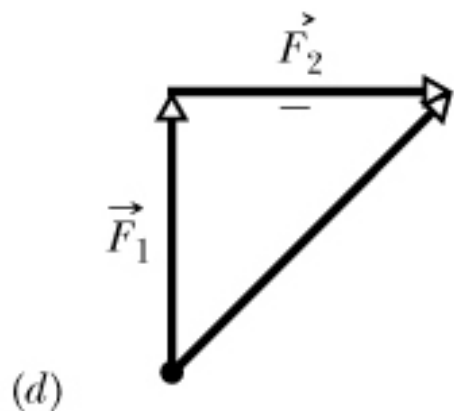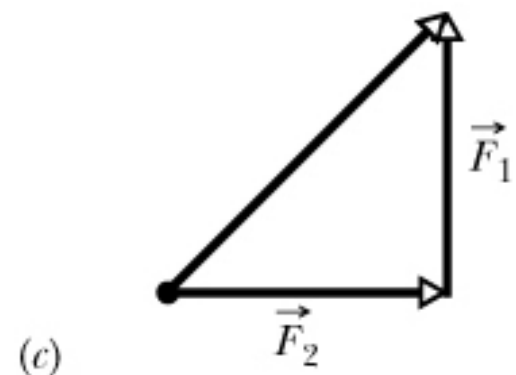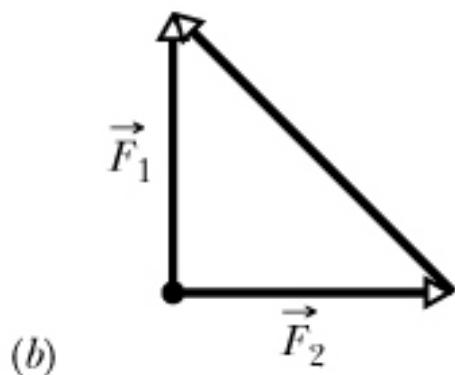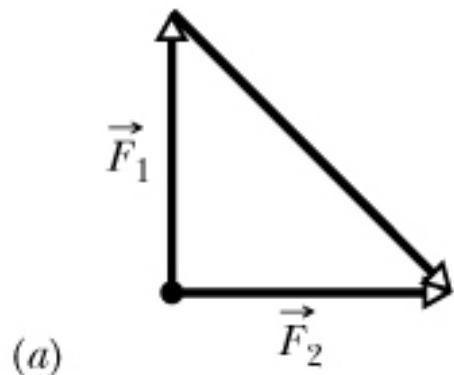
2.2. Newton's Second Law

Mass (kg): The mass of a body is the characteristic that relates a force on the body to the resulting acceleration.

> The net force on a body is equal to the product of the body's mass and its acceleration.

$$\vec{F}_{net} = m\vec{a}$$

$$F_{net,\,x} = ma_x, \quad F_{net,\,y} = ma_y, \quad F_{net,\,z} = ma_z$$

Note: If a system consists of two or more bodies, we only consider the net external force on the system to its acceleration. We do not include internal forces from bodies inside the system.

System	Force	Mass	Acceleration
SI	Newton (N)	kg	m/s²
CGS	dyne	g	cm/s²

$$1 \text{ N} = 1 \text{ kg.m/s}^2; \quad 1 \text{ dyne} = 1 \text{ g.cm/s}^2$$

To solve problems with Newton's second law, we often draw **a free-body diagram**:

- the body is presented by a dot
- each force on the body is drawn as a vector arrow with its tail on the body
- a coordinate is usually included
- you can show the acceleration of the body as a vector arrow
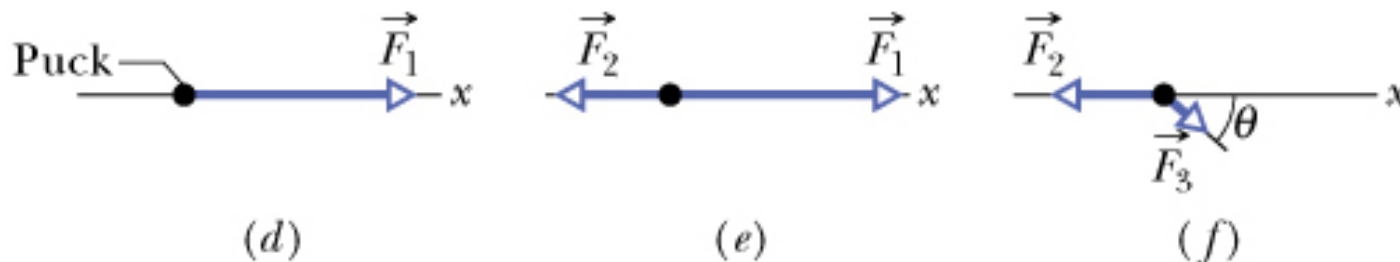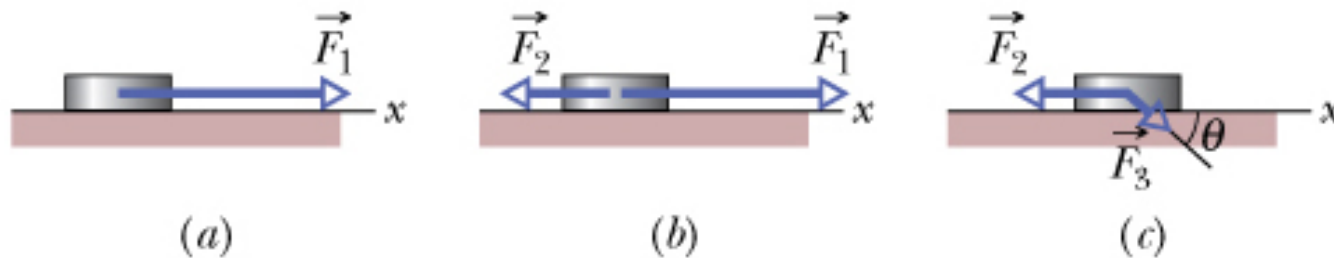
Free-body diagram

Checkpoint 2: The figure here shows two horizontal forces acting on a block on a frictionless floor. If a third horizontal force $\vec{F_3}$ also acts on the block, what are the magnitude and direction of $\vec{F_3}$ when the block is (a) stationary and (b) moving to the left with a constant speed of 5 m/s?
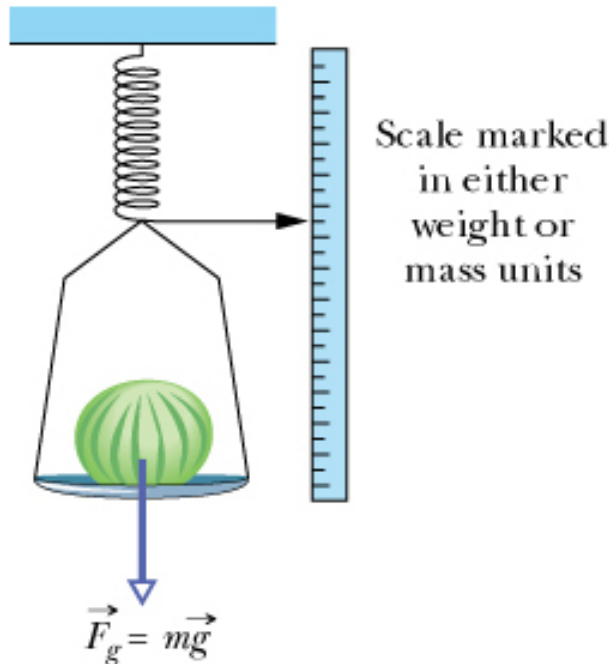
3 N   5 N

2N
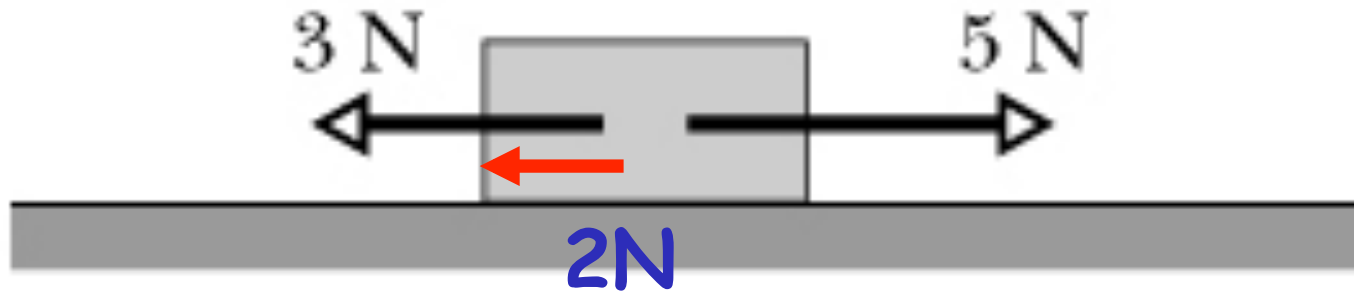
2.3. Some Particular Forces. The Gravitational Force and Weight

The gravitational force: The force of attraction between any two bodies.

$$F_g = G \frac{m_1 m_2}{r^2}$$

$G = 6.67 \times 10^{-11}$ N.m².kg⁻² is the gravitational constant.

Weight: The weight of a body is equal to the magnitude Fg of the gravitational force on the body.

Scale marked in either weight or mass units

$$W = mg$$

$$\vec{F}_g = m\vec{g}$$

Note: A body's weight is not its mass. weight depends on g.
For example, a ball of 7 kg is ~70 N on Earth but only ~12 N on the Moon since $g_{Moon} = 1.7$ m/s².
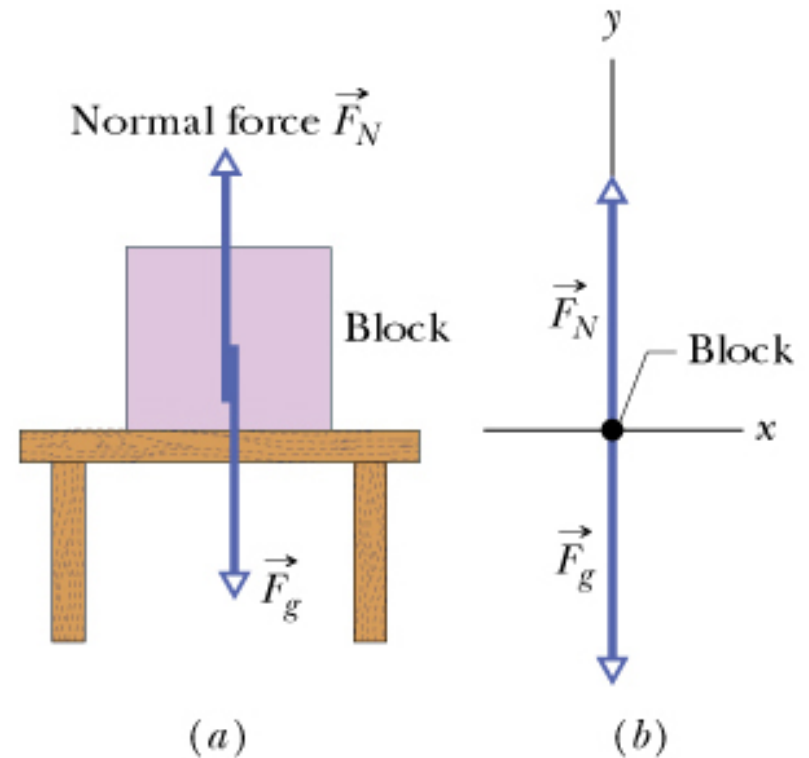
The normal force:

When a body presses against a surface, the surface deforms and pushes on the body with a normal force $\vec{F}_N$ that is perpendicular to the surface.

Example: A block of mass m presses down on a table, if the table and block are accelerating with a_y:
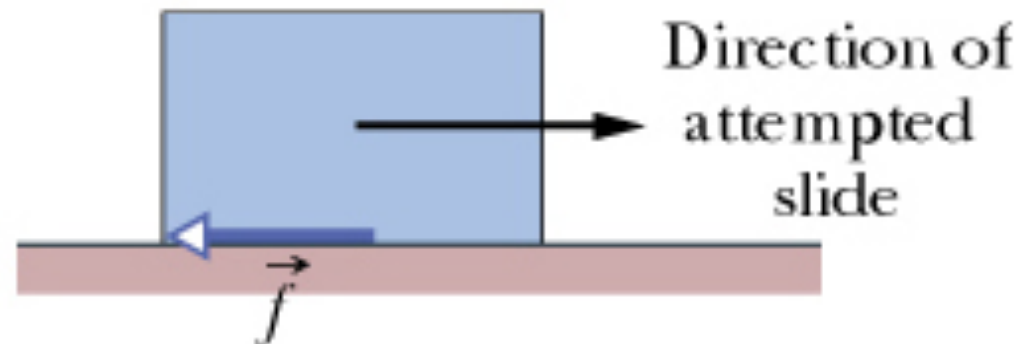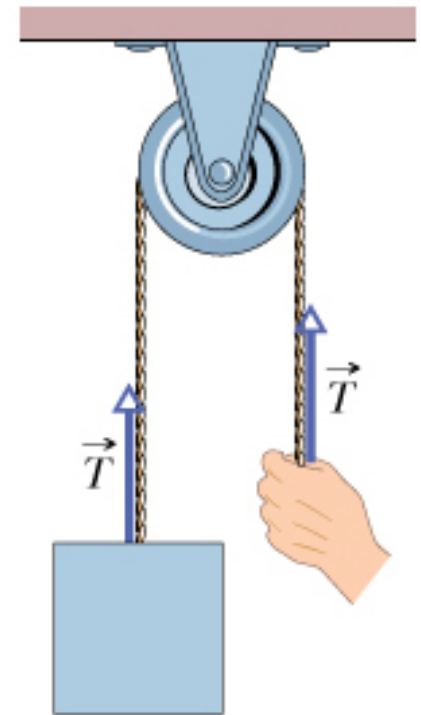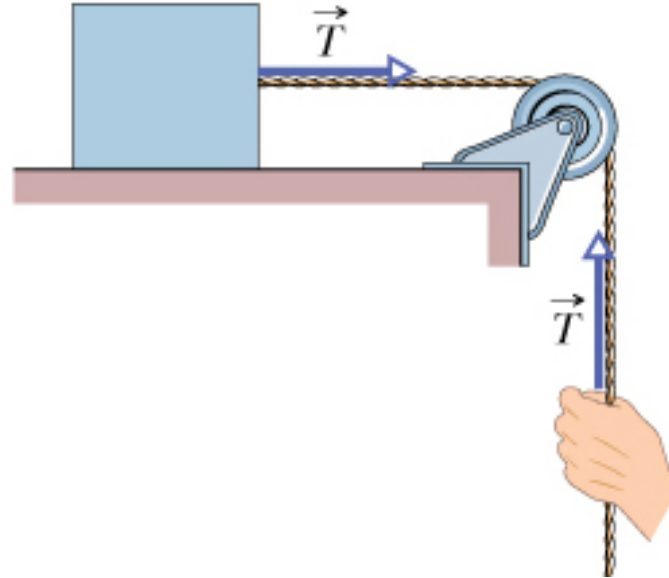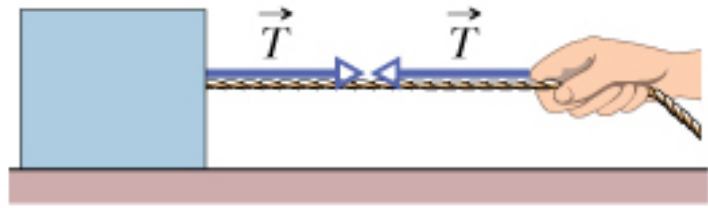
$$F_N - mg = ma_y$$

So, $\qquad\qquad F_N = m(g+a_y)$

Normal force $\vec{F}_N$

Block

$\vec{F}_g$

(a)

y

$\vec{F}_N$

Block

x

$\vec{F}_g$

(b)

Friction: The force resists the attempted slide of a body over a surface.

$\vec{f}$: frictional force

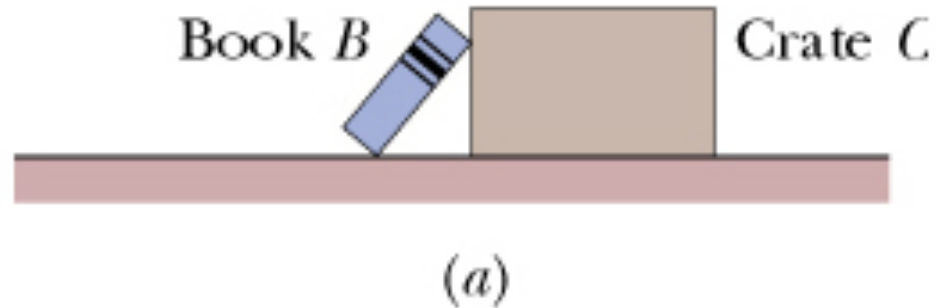Direction of attempted slide

$\vec{f}$

Tension:

$\vec{T}$: the tension force is directed away from the body and along the cord;
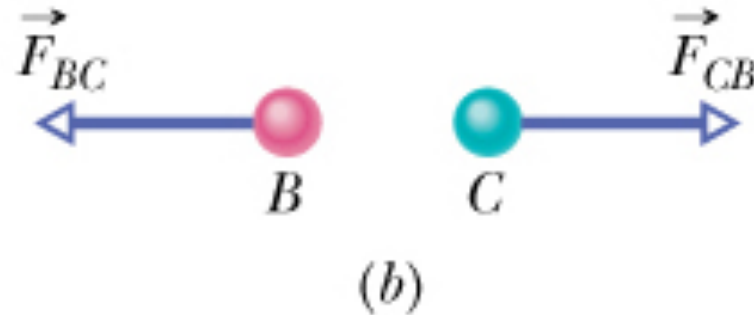
2.4. Newton's Third Law

When two bodies interact, the forces on the bodies from each other are always **equal in magnitude** and **opposite in direction**.

$$F_{BC} = F_{CB}$$

$$\vec{F}_{BC} = -\vec{F}_{CB}$$

Book B   Crate C

(a)

$\vec{F}_{BC}$ $\vec{F}_{CB}$

B   C

(b)

• The forces between two interacting bodies are called a third-law force pair.

Homework: 3, 5, 7, 13, 24, 34, 45, 49, 51, 56, 57, 59